Construction of a BRICS index and option price evaluation relative to constituent indexes

Abstract

A BRICS index is proposed relative to given indexes on the stock exchanges of the BRICS countries. The BRICS index is calculated by means of a weighted average of these given indexes. Two nonparametric models are used to construct option prices, which are converted into volatility skews via the Black-Scholes model. The first model is a relative entropy model which constructs a risk-neutral forward distribution used to price options. The second model is the Heston and Nandi model which uses a GARCH(1,1) process to construct the option prices. The effects of pricing options on the BRICS index, for different risk-free interest rates used in different countries, is discussed.

JEL Codes: C20, C63, C65, G24

Keywords: Borsa 100 index; BRICS; BRICS Index; CSI 300; FTSE/JSE Top 40; GARCH(1,1); Heston Nandi model; index; IBrX; INDEXCF; JSE; option pricing; securities exchange; S&P BSE SENSEX; volatility skew.

1. Introduction

In general, a stock market index is used to describe the performance of a sector of financial instruments traded on that stock market. Different indexes on a particular stock market provide benchmarks to compare the performance of different market sectors. Stock market indexes therefore play an important role in gauging economic performance and investment in a particular country.

BRIC was formed in 2001 and is an acronym for four developing or newly industrialised countries - Brazil, Russia, India, China. In 2010 South Africa was invited to join this association, which became known as the BRICS countries. In 2013 the BRICS countries represented nearly half the population of the world and includes the Chinese economy.
which is the second most productive economy in the world and is the fastest growing consumer market. The economic block of BRICS countries are to become a dominant supplier and consumer of goods, services and raw materials.

In this paper a BRICS index is constructed, relative to existing indexes on stock exchanges of the BRICS countries, to provide a benchmark for comparing cross border investment performance described by the constituent countries’ indexes. The construction of the cross country BRICS index will be done by using option pricing methodology.

Option price evaluation relative to the proposed BRICS index requires the use of a model. As in Hunzinger et al. [3], the Risk Neutral Historic Distribution (RNHD) model is used. For the purpose of comparison and to illustrate the model dependence and sensitivity, the the Heston and Nandi (HN) model, which uses a GARCH(1,1) process, is also used. Both models are used to generate BRICS index volatility skews for each of the BRICS constituent countries.

For each model used, this facilitates comparison and analysis of the volatility skews as these skews are generated using the same model. In general, different securities exchanges use different models to generate volatility skews and neither the method used nor the volatility skew data is made public. The shapes of volatility skews of an index on a securities exchange describe the volatility and liquidity of a local market, thus providing important information regarding the value of securities in that market. For a comprehensive account of volatility skews, the reader is referred to Kotzé et al. [10; 11]. For terminology not defined in the text, the reader is referred to Hull [9].

Hunzinger et al. [3] consider the question of comparing volatility skews of indexes on different exchanges. For the purpose of comparison, all the volatility skews on the different exchanges were generated using the same model, namely the RNHD model, and Hunzinger et al. [3] opted for a non-parametric model that has its origins in Stutzer [12], Zou & Derman [13], Buchen & Kelly [1], Duan [6] and Araújo & Maré [5]. In their study, they compared the shapes of the volatility skews of the major indexes on the BRICS securities exchanges:

- IBrX Index - top 100 stocks traded on the Bovespa, Brazil.
- INDEXCF Index - 50 most liquid Russian stocks on the Moscow Exchange.
- S&P BSE Sensex Index - is a cap weighted index on the Bombay Stock Exchange, India.
- CSI 300 Index - which consists of 300 A-shares listed on the Shanghai and Shenzhen Stock Exchanges, China.
- FTSE/JSE Top 40 Index - it includes the 40 largest companies by market capitalization on the Johannesburg Securities Exchange (JSE), South Africa.

The paper is organised as follows. The proposed BRICS index appears in Section 2. In Section 3 the model details are specified for the RNHD and the HN models. Section 4 provides the effects of interest rates on the proposed BRICS index. The volatility skews are analysed in Section 5. Section 6 concludes.
2. The proposed BRICS index

For the proposed BRICS index a weighted average is taken from monthly values (in US dollars) of domestic market capitalization from major stock exchanges of the BRICS countries.

The total value (in US dollars) of the domestic market capitalization from the respective stock exchanges for each month from 31 March 2009 until 31 April 2014 is determined and the percentage of domestic market capitalization for each country is calculated. The percentage of each countries monthly domestic market capitalization is then used to weight the relevant daily index price. The BRICS index for day $t_i$ is defined as

$$BRICS(t_i) = \sum_{j=1}^{5} W(t_i, C_j) I(t_i, C_j),$$

where $W(t_i, C_j)$ denotes the weighting of country $C_j$ for the month containing the date $t_i$, and $I(t_i, C_j)$ denotes the index at time $t_i$ from country $C_j$ which is used to calculate the BRICS index. The stock exchanges which are constituents of the weighted average domestic trade value for each of the BRICS members are given below:

- Russia: MICEX, Moscow Exchange.
- India: BSE India, National Stock Exchange India.
- China: Hong Kong -, Shanghai- and Shenzhen Stock Exchange.

It is important to note that the Russian stock exchange MICEX stopped operating at the end of 2011, and the operations were taken over by the Moscow exchange at the beginning of 2012.

3. Model specification

3.1. Relative entropy model

The first model that we consider was also used in Hunzinger et al. [3]. Some of the details presented in [3] are repeated here for the convenience of the reader.

The model is based on the principle of relative entropy, with which a change of measure transforms a real world returns distribution of an asset into a risk-neutral returns distribution, which can then be used to find the price of an option by discounting the future expected cash flow under the risk-neutral measure by the risk-free rate.

For a given set of historical closing prices $M$ we obtain the daily $T$–period returns using

$$\text{return}_i = \frac{\text{index}_{i+T}}{\text{index}_i} \quad \text{for} \quad i = 1, ..., M - T,$$

where $T$ is the maturity of the option to be valued.
We construct a historical return distribution by splitting the returns into bins of equal length and calculating the cumulative sum of the number of observations that lie in each bin. We divide this sum by the total number of observations to obtain the historical return distribution $P$.

To obtain the risk-neutral historical probability distribution (RNHD) $Q$, we apply the relative entropy principle which states that there exists a distribution $Q$ such that $E_Q[S_T] = S_0 \exp[rT]$, and

$$E_Q \left[ \log \left( \frac{Q(x)}{P(x)} \right) \right] = \min \left\{ E_R \left[ \log \left( \frac{R(x)}{P(x)} \right) \right] : R \text{ is a distribution and } E_R[S_T] = S_0 \exp(rT) \right\},$$

where $S_0$ is the spot price of the index, $S_t$ is the price at time $t$ and $r$ is the risk-free rate.

The RNHD $Q$ is calculated by applying the theory of Lagrange multipliers, which states that there exists a $\psi$ such that

$$Q(x) = \frac{P(x) \exp(\psi x)}{\int_{-\infty}^{\infty} P(x) \exp(\psi x) \, dx},$$

and $\psi$ is determined numerically (see Cover and Thomas [4]). Under the risk-neutral measure $Q$ the price of an option is then calculated by discounting the future expected payoff by the risk-free rate. The price of a European call option with strike $K$ and maturity $T$ is then given by

$$C_0 = \frac{\sum_x Q(x) \max(S_0 \exp rT \text{return}_x - K, 0)}{e^{rT}}.$$

3.2. Heston and Nandi model

The second model, proposed by Heston and Nandi (HN) in [2], suggests using a GARCH($p,q$) process to model the variance of the spot price asset $S$. The model uses historical stock price data to simultaneously capture the correlation of volatility with spot returns and the path dependency in volatility. Two assumptions are made in order to implement this model, namely that over a time period of length $\Delta$, the spot asset price follows a process given by

$$R_t = \log \left( \frac{S_t}{S_{t-1}} \right) = r + \lambda h_t + \sqrt{h_t} z_t,$$

where

$$h_t = \beta^{(0)} + \sum_{i=1}^{p} \beta^{(1)}_{i} h_{t-\Delta_i} + \sum_{i=1}^{q} \beta^{(2)}_{i} \left( z_{t-\Delta_i} - \gamma_i \sqrt{h_{t-\Delta_i}} \right)^2,$$

and $r$ equals the continuously compounded risk-free rate, and $z$ is a standard normal random variable. We set $p = q = 1$ in order to form a GARCH(1,1) model. Then $h_t$ is the conditional variance of log returns between time $t - \Delta$ and $t$, and has a filtration set at time $t - \Delta$.

To calculate the price of an European option, we discount the future expected payoff by the risk-free rate. In order to do so, the risk-neutral distribution of the spot price
needs to be found – in the Heston and Nandi model the GARCH(1,1) risk-neutral asset spot price process is given by
\[ R^*_t = \log \left( \frac{S_t}{S_{t-\Delta}} \right) = r - \frac{1}{2} h_t + \sqrt{h_t} z^*_t, \]
where\[
h_t = \beta^{(0)} + \beta^{(1)}_1 h_{t-\Delta} + \beta^{(2)}_1 \left( z^*_{t-\Delta} - \gamma^*_1 \sqrt{h_{t-\Delta}} \right)^2,\]
\[ z^*_t = z_t + \left( \lambda + \frac{1}{2} \right) \sqrt{h_t}, \]
\[ \gamma^*_1 = \gamma_1 + \lambda + \frac{1}{2}. \]
In addition to the risk-neutral distribution of the spot price process, the generating function of the GARCH process is needed to value options. If \( f(\phi) \) denotes the conditional generating function of the asset price, i.e.
\[ f(\phi) = \mathbb{E}_t [S_T^\phi] = \mathbb{E}[S_T | F_t], \]
then the generating function takes the log-linear form given by
\[ f(\phi) = S_t^\phi \exp \left[ A_t + B_t h_{t+\Delta} \right], \]
where\[
A_t = A_{t+\Delta} + \phi r + B_{t+\Delta} \beta^{(0)} - \frac{1}{2} \ln \left( 1 - 2\beta^{(2)} B_{t+\Delta} \right),
\]
\[ B_t = \phi (\lambda + \gamma_1) - \frac{1}{2} \gamma_1^2 + \beta^{(1)} B_{t+\Delta} + \frac{1}{2} \frac{(\phi - \gamma_1)^2}{1 - 2\beta^{(2)} B_{t+\Delta}}. \]
These coefficients can be calculated recursively from the terminal conditions of \( A \) and \( B \) given by
\[ A_T = B_T = 0. \]
The generating function of the spot price is the moment generating function of the logarithm of the spot price, and so \( f(i\phi) \) is the characteristic function of the logarithm of the spot price, where \( i = \sqrt{-1} \). Then the price of an European call option can be valued as
\[ C = e^{-rT} \mathbb{E}_0^* [(S_T - K)^+] \]
\[ = \frac{1}{2} S_t + \frac{e^{-rT}}{\pi} \int_0^\infty \text{Re} \left[ K^{-i\phi} f^*(i\phi + 1) \right] d\phi \]
\[ - K e^{-rT} \left( \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \text{Re} \left[ K^{-i\phi} f^*(i\phi) \right] d\phi \right), \]
where the expectation is taken under the risk-neutral distribution.

To find the option prices using the HN model, five parameters need to be calibrated to the historical returns of the individual BRICS countries indexes, namely $\beta^{(0)}$, $\beta^{(1)}$, $\beta^{(2)}$, $\lambda$ and $\gamma$. First, the daily log returns are found from the historical closing prices, of which the conditional variance is found for each time point. We then find the parameters of the GARCH(1,1) process using maximum likelihood estimators.

4. Volatility skews of the models

The central bank rates of the various BRICS nations are used as a proxy for the risk-free rate, and are given in Table 1, along with the relevant spot index levels on the 28th May 2014. The fitted parameters are given in Table 2.

Table 1: The central bank rate for the various BRICS countries and the relevant countries’ index spot levels on the 28th May 2014. (Source: [7; 8]).

<table>
<thead>
<tr>
<th>BRICS nation</th>
<th>Central bank rate</th>
<th>Index</th>
<th>Current index level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil</td>
<td>11%</td>
<td>IBrX</td>
<td>21695.41</td>
</tr>
<tr>
<td>Russia</td>
<td>7%</td>
<td>INDEXCF</td>
<td>1381.50</td>
</tr>
<tr>
<td>India</td>
<td>8%</td>
<td>S&amp;P BSE Sensex</td>
<td>24556.09</td>
</tr>
<tr>
<td>China</td>
<td>6%</td>
<td>CSI 300</td>
<td>2169.35</td>
</tr>
<tr>
<td>South Africa</td>
<td>5.5%</td>
<td>FTSE/JSE Top 40</td>
<td>44732.26</td>
</tr>
</tbody>
</table>

Using historical closing prices from the first trading date in 2009 up until the 28th May 2014 we obtain the parameters given in Table 2 for the different indexes under the HN model.

Table 2: HN parameters for the different indexes of the BRICS countries, calibrating to historical closing prices from the first trading date in 2009 until the 28th May 2014.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$\beta^{(0)}$</th>
<th>$\beta^{(1)}$</th>
<th>$\beta^{(2)}$</th>
<th>$\lambda$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>IBrX</td>
<td>1.1937e-15</td>
<td>0.9425</td>
<td>7.3351e-06</td>
<td>-35.7775</td>
<td>47.6875</td>
</tr>
<tr>
<td>INDEXCF</td>
<td>7.9356e-14</td>
<td>0.9398</td>
<td>1.4031e-05</td>
<td>-14.9878</td>
<td>13.7570</td>
</tr>
<tr>
<td>CSI 300</td>
<td>1.3065e-07</td>
<td>0.2170</td>
<td>2.6689e-10</td>
<td>-15.0707</td>
<td>0.2820</td>
</tr>
<tr>
<td>FTSE/JSE Top 40</td>
<td>6.5876e-19</td>
<td>0.9188</td>
<td>7.5084e-06</td>
<td>-20.9600</td>
<td>56.8016</td>
</tr>
</tbody>
</table>

To investigate whether the HN and RNHD option pricing models can reproduce market prices, we compare the market call option prices obtained by the models to market data published by the JSE. The options are written on the FTSE/JSE Top 40 index, and we use the interest rates and spot prices given in Table 1, and the fitted parameters given in Table 2. Figure 1 demonstrates how the models reproduce the market call option prices over a range of strike prices. The HN model reproduces the market prices more accurately than the RNHD model, since the RNHD model is generally overpricing the option.

The implied volatility of the call option prices is found through the Black-Scholes closed form solution for option prices. In figure 2 we can see that both option pricing
models reproduce the characteristic implied volatility skew found in the JSE published market implied volatility. The RNHD - and HN model have a greater implied volatility further out-of-the-money and in-the-money. The implied volatility skew is approximately the same for at-the-money options. The RNHD implied volatility is generally higher than the HN skew, which reflects that the RNHD model overprices options that lie further out-of-the-money or in-the-money.
4.1. Effects of interest rates of each country

The BRICS index proposed in this paper is constructed through a weighted average. In order to price options with the RNHD method or the HN model presented previously, we use the interest rates of each respective country in order to determine the forward price distribution of the underlying index. The question of which interest rate to use for the BRICS index arises, since it’s constituents are made up from several countries.

The answer to this question is as follows: options on the BRICS index will have different prices and implied volatilities depending on the country in which the option is written. If an option is written on the BRICS index in Russia, then the forward distribution of the BRICS index will be calculated using the Russian central bank rate, i.e. 7%. If however the option is written in South Africa, then the forward distribution of the BRICS index will be calculated using the South African central bank rate of 5.5%. From figure 3 and 4 we can see that a higher domestic central bank rate (which we used as a proxy for the risk-free rate) causes the implied volatility, and therefore the price of an option, to be greater. This means that Brazil will generally price an option on the BRICS index more expensive than, for example, Russia, since they have a higher interest rate.

Even though different countries price the same option at different levels, and therefore at different implied volatilities, this does not allow any arbitrage opportunities to occur, since an investor will still be faced with the same opportunity cost involved by needing to borrow and invest at different interest rates.

Figure 3: The RNHD model implied volatility skews for different countries for an European call option on the BRICS index from 28th May 2014 with expiry 19th June 2014. The RNHD model is calibrated to historical closing prices from 2nd January 2009 to 28th May 2014.
In table 3 we show the calibrated parameters from the HN model from the perspective of writing the option in each individual BRICS country. Since the interest rates are different, we can see how the calibrated GARCH parameters change.

<table>
<thead>
<tr>
<th>Country</th>
<th>Interest rate</th>
<th>$\beta^{(0)}$</th>
<th>$\beta^{(1)}$</th>
<th>$\beta^{(2)}$</th>
<th>$\lambda$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>South Africa</td>
<td>0.05</td>
<td>3.63E-36</td>
<td>0.8476</td>
<td>1.13E-05</td>
<td>-39.3922</td>
<td>-0.4043</td>
</tr>
<tr>
<td>China</td>
<td>0.06</td>
<td>2.14E-35</td>
<td>0.8235</td>
<td>1.31E-05</td>
<td>-43.4444</td>
<td>-0.3856</td>
</tr>
<tr>
<td>Russia</td>
<td>0.07</td>
<td>9.05E-34</td>
<td>0.6744</td>
<td>2.42E-05</td>
<td>-51.5490</td>
<td>-0.3483</td>
</tr>
<tr>
<td>India</td>
<td>0.08</td>
<td>1.67E-31</td>
<td>0.4821</td>
<td>3.85E-05</td>
<td>-59.6535</td>
<td>-0.3118</td>
</tr>
<tr>
<td>Brazil</td>
<td>0.11</td>
<td>2.10E-24</td>
<td>0.2070</td>
<td>5.90E-05</td>
<td>-83.9671</td>
<td>-0.2048</td>
</tr>
</tbody>
</table>

5. Comparison of the BRICS countries’ volatility skews

Using the RNHD and the HN model it is possible to compare volatility skews using only the historical closing prices for indexes across the BRICS countries. We use the indexes given in table 1 and the relevant interest rates to produce option prices for the different indexes, and then find the implied volatility skew for each index, as well as for the proposed BRICS index. The BRICS index will be written from the perspective of an South African investor.

In both models the South African FTSE/JSE Top 40 index has the lowest implied volatility. Under the HN model the FTSE/JSE Top 40 index and the Brazilian IBrX index are almost perfectly parallel, where as under the RNHD model the two skews assume the same shape, but are not parallel. The IBrX has the same characteristics for both models – it has the greatest implied volatility as the moneyness becomes less. Under the HN
RNHD model: implied volatility skews

Figure 5: The RNHD model implied volatility skews for the different countries' indexes and the BRICS index for an European call option from 28th May 2014 with expiry 19th June 2014. The model is calibrated to historical closing prices from 2nd January 2009 to 28th May 2014, and the BRICS index option is valued from an South African perspective.

HN model: implied volatility skews

Figure 6: The HN model implied volatility skews for the different countries’ indexes and the BRICS index for an European call option from 28th May 2014 with expiry 19th June 2014. The model is calibrated to historical closing prices from 2nd January 2009 to 28th May 2014, and the BRICS index option is valued from an South African perspective.

model the Russian INDEXCF index and the Chinese CSI 300 index skews are almost identical, where as under the RNHD model they are significantly different.
In the RNHD model the proposed BRICS index implied volatility skew lies almost perfectly on top of the FTSE/JSE Top 40 index implied volatility skew for high moneyness levels, and tend to increase as the moneyness becomes lower.

In the HN model the proposed BRICS index implied volatility skew is almost perfectly identical to the FTSE/JSE Top 40 index implied volatility skew.

6. Conclusion

A BRICS index was constructed using a weighted average of the domestic market capitalization from indexes on the major stock exchanges from the BRICS countries. We compared the implied volatility skews of the RNHD and the HN models to the market implied volatility skews on the FTSE/JSE Top 40 index. The HN model reproduced the market implied volatility skew more accurately than the RNHD model. This can be attributed to the fact that the HN model has five calibration parameters, where as the RNHD model only has one. The RNHD and the HN model do however produce similar results, but they show different sensitivities to data selection and interest rates. The prices of options on the BRICS index were constructed and converted into volatility skews. The level of the volatility skew of the BRICS index depends on which country is pricing the option, as the option price is dependent on the risk-free rate of each specific country.

The methods used for the proposed index and the volatility skew analysis presented in this paper are general and could be applied more widely. For example, application to an ASEAN index, for a given set of representative indexes from each ASEAN country, is possible.

References


